Math 233H

people.math.umass.edu/~gunnells/calc/calc.html

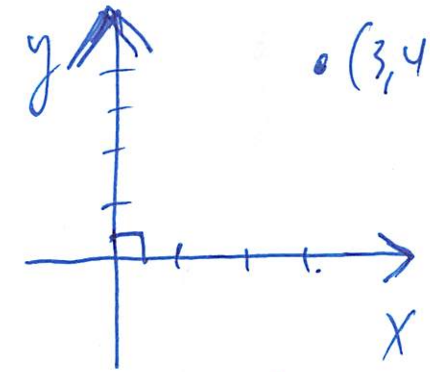
- syllabus
- scans of lectures
- resources / info

Office hours TBA

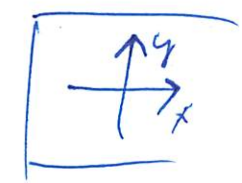
TA Dan Gallagher.
Tues 4 PM.

§ 12.1 3D Coordinates.

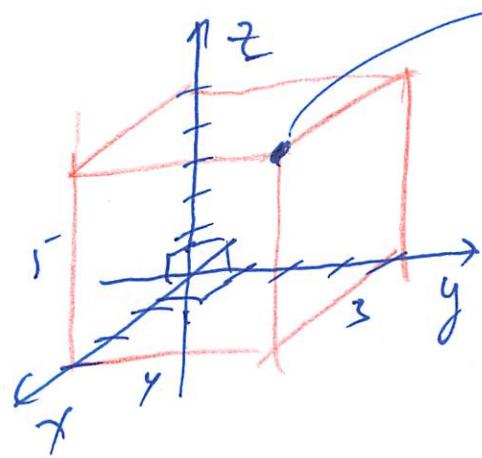
2D coords. (x, y)



$(3, 4)$

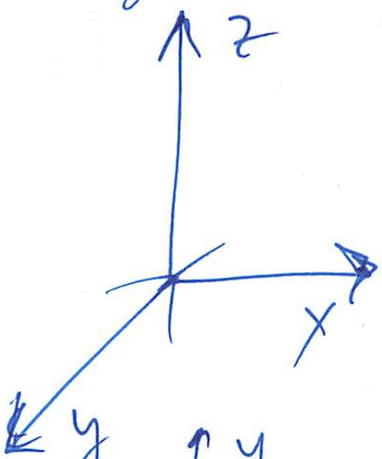


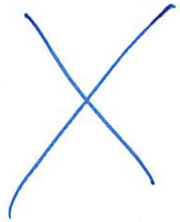
3D cartesian coords

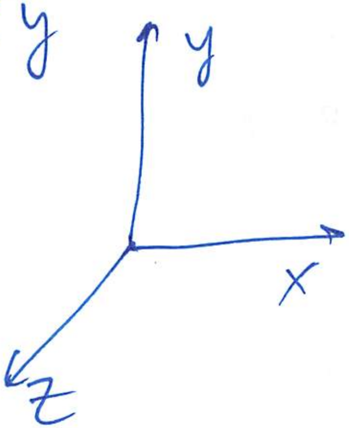


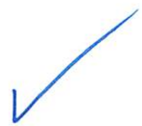
$(3, 4, 5)$.
x y z

we chose labels according to right hand rule.





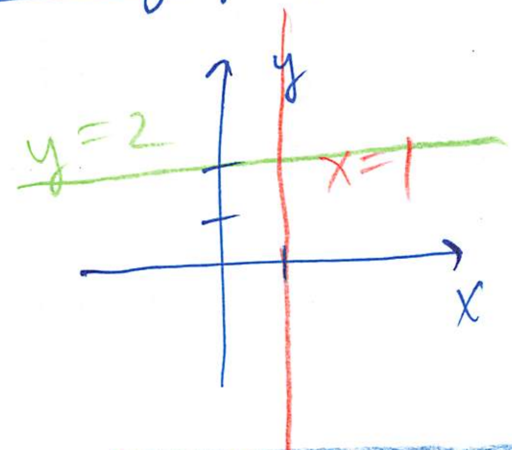




RH rule: use fingers of RH to push x-axis into y-axis. RH Thumb points in direction of z-axis

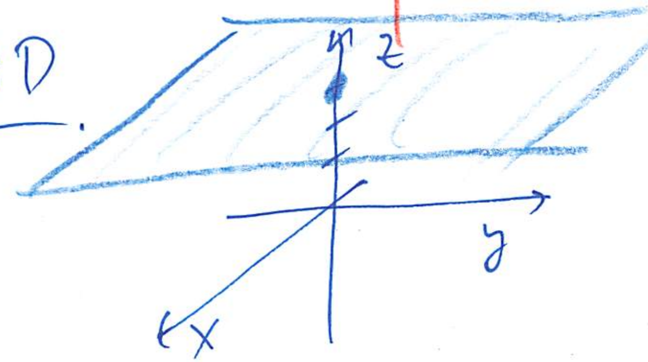
Simple graphs

2D



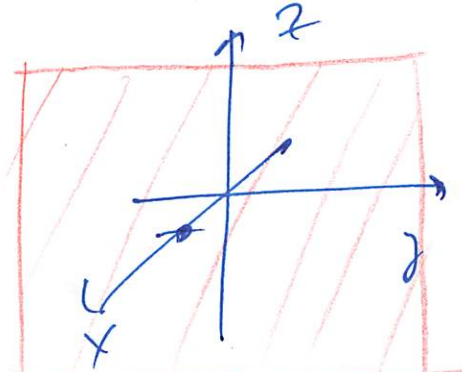
$x = 1$
$y = 2$

3D.



$z = 3$

plane at height 3 above xy plane



$x = 1$

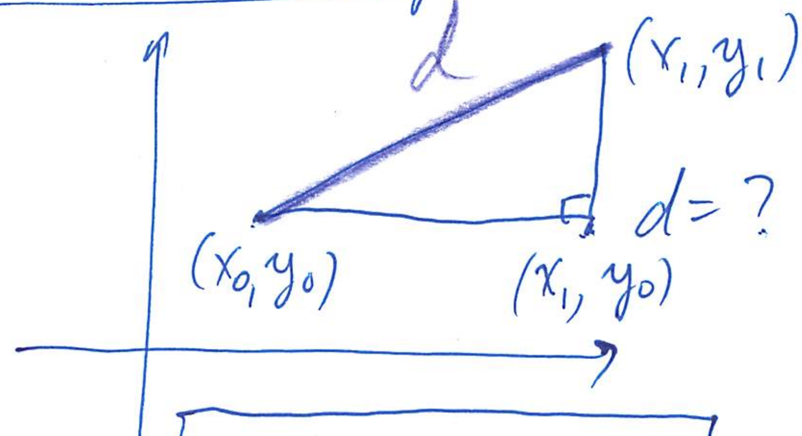
$z=0 \iff xy$ coord plane

$y=0 \iff xz$ " "

$x=0 \iff yz$ " "

distance formula

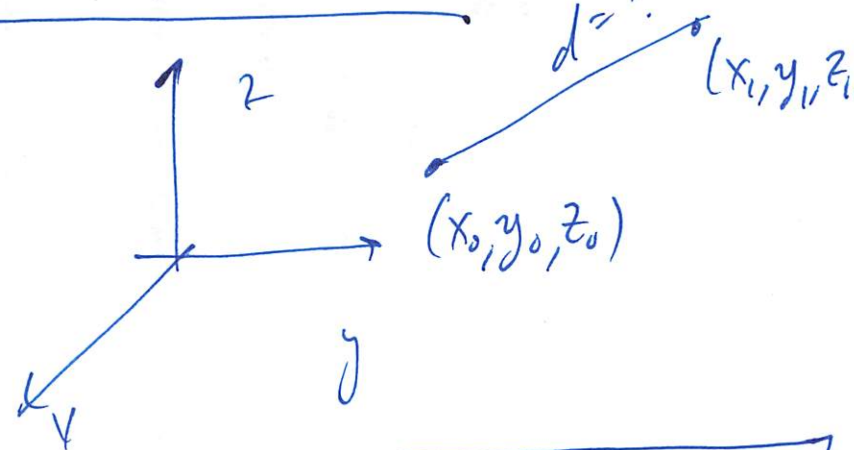
2D distance formula.



$$d=\sqrt{(x_0-x_1)^2+(y_0-y_1)^2}$$

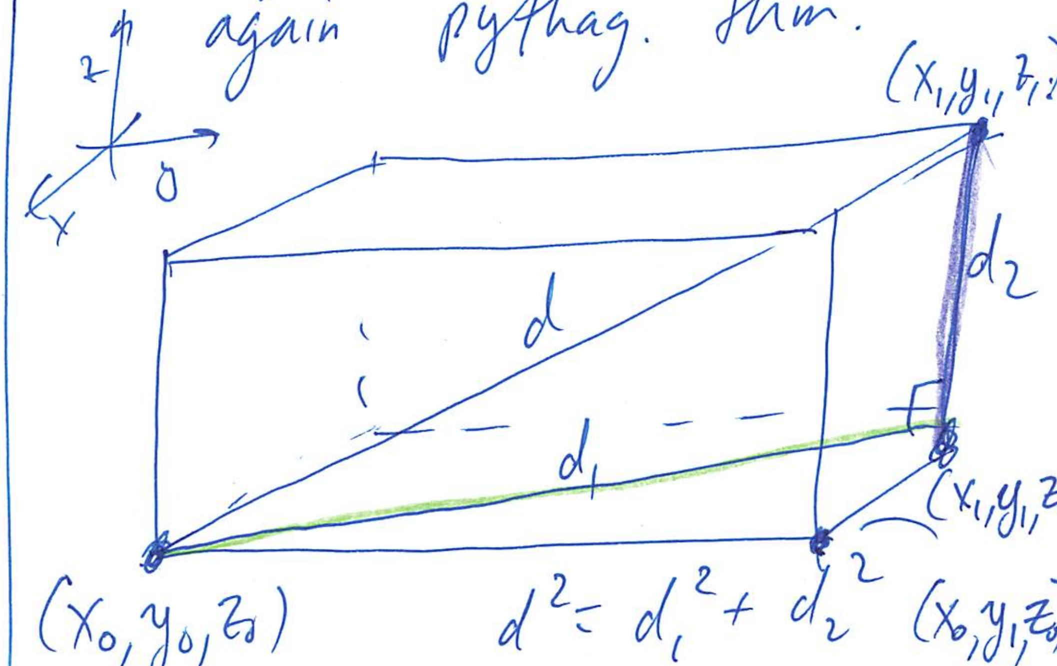
manifestation of Pythag. Thm

3D distance formula



$$d=\sqrt{(x_0-x_1)^2+(y_0-y_1)^2+(z_0-z_1)^2}$$

again pythag. thm.



$$d^2=d_1^2+d_2^2$$

$$d_1 = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}$$

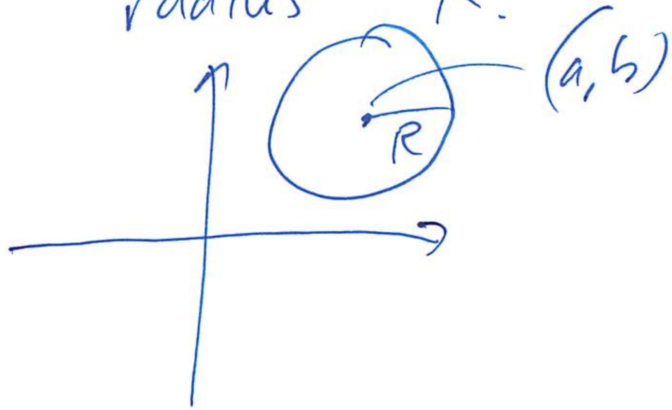
$$d_2 = \sqrt{(z_0 - z_1)^2}$$

$$\Rightarrow d = \sqrt{\ldots\ldots}$$

what we want

circles / spheres.

2D center (a,b)
radius R.



$$\Rightarrow R = \sqrt{(x-a)^2 + (y-b)^2}$$

or $(x-a)^2 + (y-b)^2 = R^2$

e.g. unit circle @ origin.

$$x^2 + y^2 = 1$$

3D sphere.
center (a,b,c)
radius R

$$\boxed{\text{eqn}: (x-a)^2 + (y-b)^2 + (z-c)^2 = R^2}$$

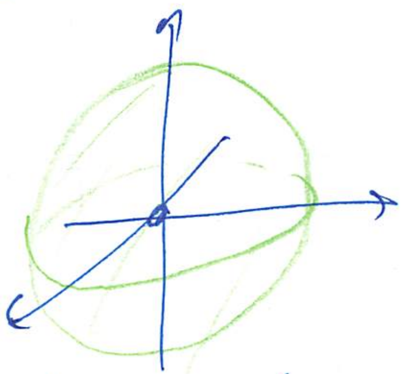
T

e.g. unit sphere @ origin

center (0,0,0)

Radius = 1.

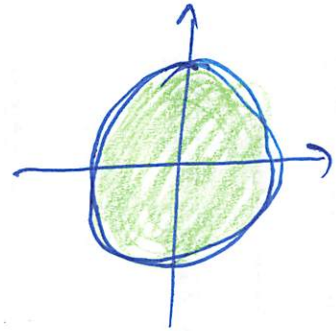
$x^2+y^2+z^2=1$



just the surface, not the points inside.

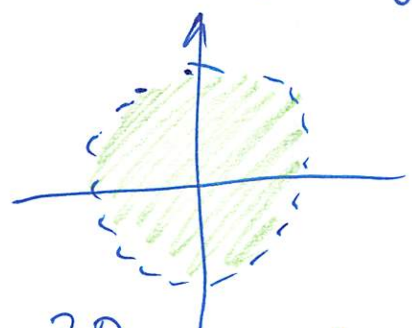
e.g. 2D $x^2+y^2 \le 1$

closed unit disk at origin



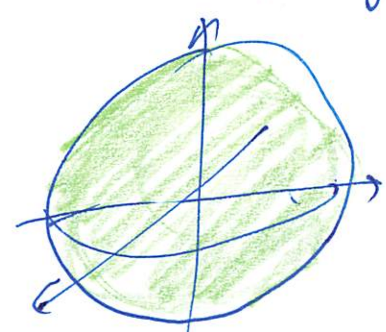
e.g. open unit disk

$x^2+y^2<1$



don't include edge.

e.g. 3D $x^2+y^2+z^2 \le 1$



now includes pts inside.

§12.2 Vectors

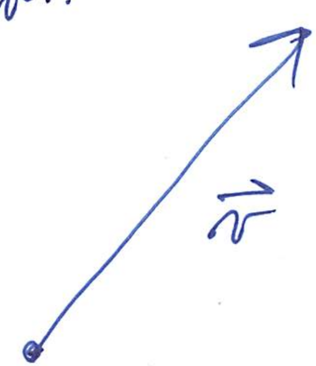
vector = object that has both magnitude and direction.

scalar = object only with magnitude, not direction. (aka. number)

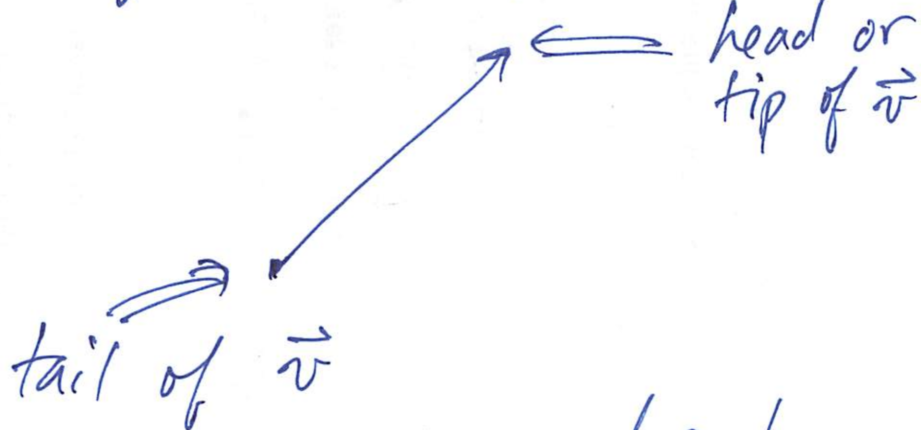
We represent vectors by arrows.

$\vec{V}$

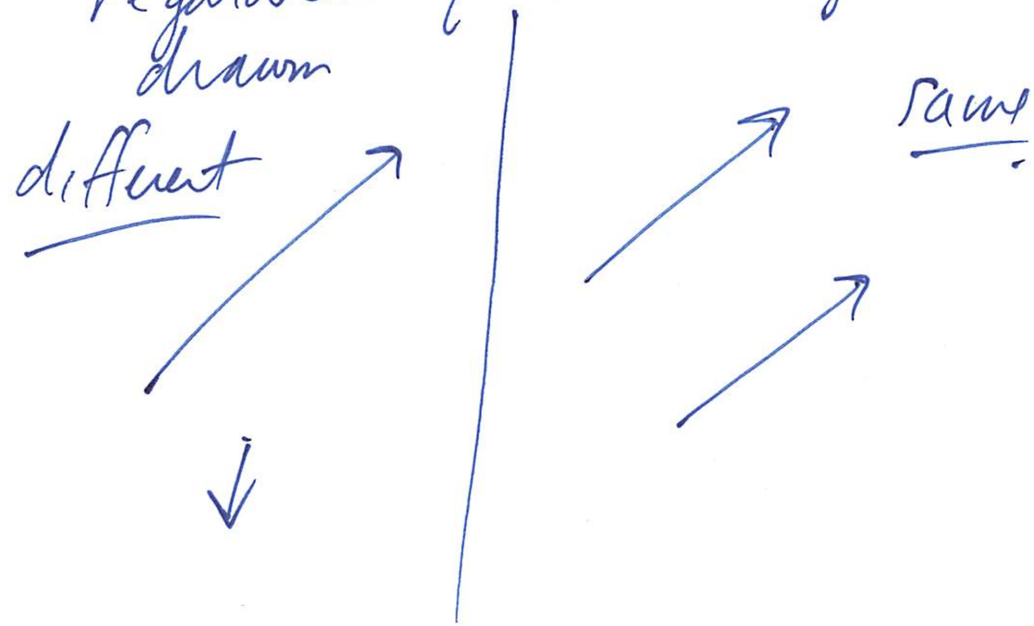
(In text use boldface **v**)



magnitude = length.

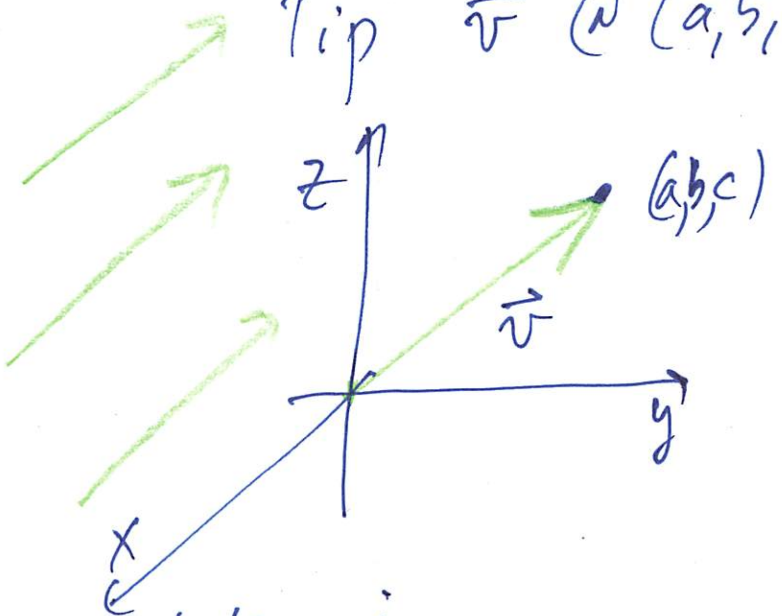


we consider 2 vectors to be the same if have length and same direction, regardless of where they are drawn



Connection between points in 3D and vectors

$P = (a,b,c)$ in 3D.
it determines a vector $\vec{v}$. Tail $\vec{v}$ @ origin
Tip $\vec{v}$ @ (a,b,c)



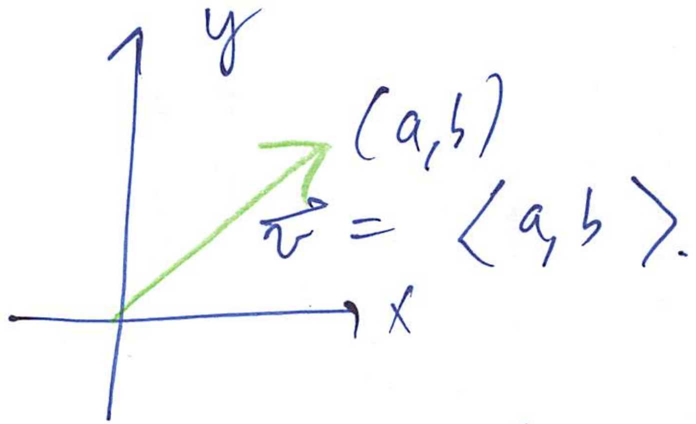
notation is
$\vec{v} = \langle a,b,c \rangle$.

so a pt determines a vector. Conversely, a vector determines a pt.
given $\vec{v}$, ~~pt~~ put tail of $\vec{v}$ @ origin.
The tip is then a pt in 3D.

$$\vec{v} = \langle a,b,c \rangle \Longrightarrow P = (a,b,c)$$

Remarks

① can also do this in 2D.



② If $\vec{v} = \langle a, b, c \rangle$, these numbers are called the <u>components</u> of $\vec{v}$.

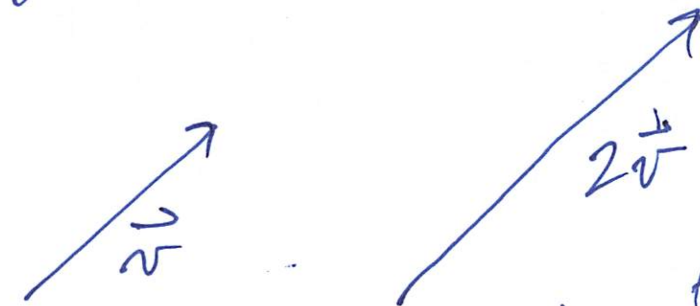
<u>Operations on vectors</u>.

① can multiply a vector by a scalar

$\vec{v}$ vector

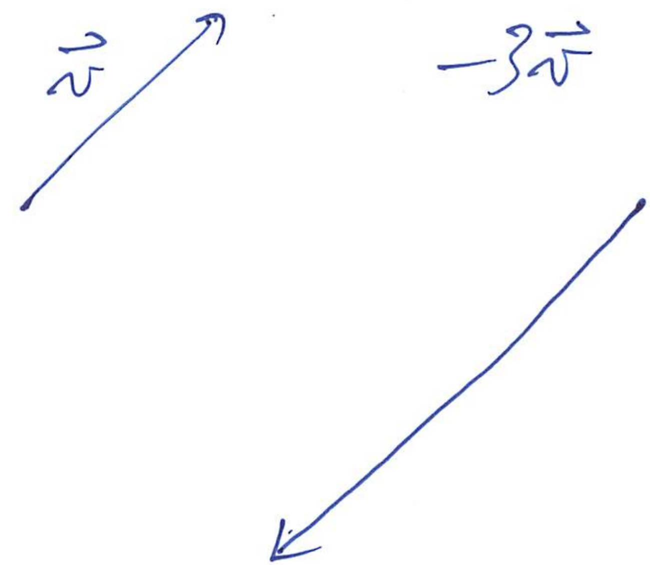
c scalar:

get new vector $c\vec{v}$: ①



just the vector by factor of c.

If scalar is negative, we make the new vector point in opposite direction.



In terms of components,
spose $\vec{v} = \langle a, b, c \rangle$
scalar α

Then $\alpha \vec{v} = \langle \alpha a, \alpha b, \alpha c \rangle$.

i.e. just scale the components

Think through why this is the same.

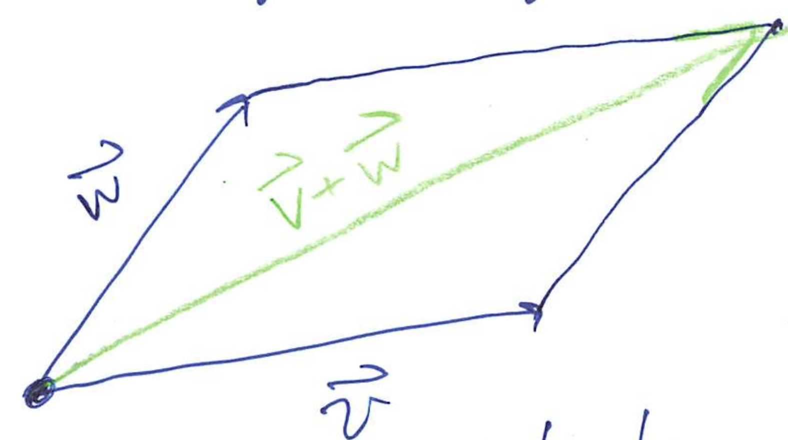
2. Can add vectors.

$\vec{v}, \vec{w} \rightsquigarrow$ "$\vec{v} + \vec{w}$"

Geometric picture: parallelogram rule

place $\vec{v}, \vec{w}$ with their tails at the same point.

Build parallelogram with edges along $\vec{v}, \vec{w}$.



$\vec{v} + \vec{w}$ is defined to be the vector going across the diagonal.

Algebraic picture: add components.

$\vec{v} = \langle a_1, b_1, c_1 \rangle$

$\vec{w} = \langle a_2, b_2, c_2 \rangle$

$\vec{v} + \vec{w} = \langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle$

$\vec{v} + \vec{w} = \vec{w} + \vec{v}$

(3) subtraction.

$\vec{v} - \vec{w} = ?$

Def $\vec{v} + (-1)\vec{w}$

components:

$\vec{v} - \vec{w} = \langle a_1 - a_2, b_1 - b_2, c_1 - c_2 \rangle$

$\vec{v} - \vec{w} =$

$-(\vec{w} - \vec{v})$

(4) can take the length of $\vec{v}$.

notation: $|\vec{v}|$

or $\|\vec{v}\|$

$\vec{v} = \langle a, b, c \rangle$

$|\vec{v}| = \sqrt{a^2 + b^2 + c^2}$

comes from distance formula

$$|\alpha \vec{v}| = |\alpha|\,|\vec{v}|$$

$$|\vec{v}| \geq 0$$

only vector with length 0 is $\langle 0,0,0 \rangle$.

$\vec{0}$ "zero vector"

$$\vec{v} + \vec{0} = \vec{0} + \vec{v} = \vec{v}$$

<u>Def</u> a vector is a unit vector if its length is 1.

$$|\vec{v}| = 1.$$

e.g.

$$\langle 1,0,0 \rangle = \hat{i}$$

$$\langle 0,1,0 \rangle = \hat{j}$$

$$\langle 0,0,1 \rangle = \hat{k}$$

3 special unit vectors

$\hat{i}$ points along x-axis

$\hat{j}$ - - - - - - y-axis

$\hat{k}$ - - - - - - - - z-axis

(usually use ^ instead of $\rightarrow$ for unit vectors)

$\vec{v}$ vector, no assumption about length.

$\hat{v}$ $|\hat{v}| = 1$.

e.g. Find a unit vector in same direction as $\vec{v} = \langle 2, 3, 6 \rangle$.

ANS: take vector $\hat{v} = \frac{1}{|\vec{v}|} \vec{v}$

$$|\vec{v}| = \sqrt{2^2 + 3^2 + 6^2}$$
$$= \sqrt{4 + 9 + 36}$$
$$= \sqrt{49} = 7$$

$$\hat{v} = \frac{1}{7} \langle 2, 3, 6 \rangle$$
$$= \left\langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \right\rangle.$$