DISCLAIMER: This practice exam is intended to give you an idea about what a two-hour final exam is like. It is not possible for any practice exam to cover every topic, and the content, coverage and format of your actual exam could be different from this practice exam.

Part I: Multiple Choice Problems. You only need to give the answer; no justification needed.

1. Describe the surface whose equation in cylindrical coordinates is $z = 4r$.
   (a) cylinder with vertical axis (b) cylinder with horizontal axis (c) sphere
   (d) half cone with vertical axis (e) half cone with horizontal axis (f) plane

2. Which of the following double integrals are such that reversing the order of integration would result in two double integrals?
   A: $\int_{-1}^{2}\int_{x^2-2}^{x} f(x,y)\,dy\,dx$, **B:** $\int_{0}^{1}\int_{y^2}^{2-y} g(x,y)\,dx\,dy$, **C:** $\int_{0}^{1}\int_{\arctan x}^{\pi/4} h(x,y)\,dy\,dx$
   (a) A only (b) B only (c) C only (d) A & B (e) A & C (f) B & C

3. Which of the following points is a local maximum of the function $f(x,y) = xy - x^2y - xy^2$?
   (a) $(0,0)$ (b) $(1,1)$ (c) $(0,2)$ (d) $(2,0)$ (e) $(1/3,1/3)$ (f) $(1/2,1/2)$

4. Determine the set on which the function $f(t) = \dfrac{\sqrt{t}-1}{\sqrt{t}+1}$ is continuous.
   (a) $t > 0$ (b) $t \geq 0$ (c) t >1 (d) $t \geq 0$ and $t \neq 1$ (e) $t \neq 0$ (f) all real numbers

5. Let $f(x,y,z)$ be a differentiable function, and let $\vec{F}(x,y,z)$ be a differentiable, 3-dimensional vector field. Which of the following formulae is correct?
   (a) $\mathrm{div}(f\vec{F}) = f\mathrm{div}\vec{F} + \mathrm{curl}\vec{F} \bullet \nabla f$ (b) $\mathrm{div}(f\vec{F}) = f\mathrm{curl}\vec{F} + \vec{F} \times \nabla f$
   (c) $\mathrm{curl}(f\vec{F}) = f\mathrm{div}\vec{F} + \vec{F} \bullet \nabla f$ (d) $\mathrm{div}(f\vec{F}) = f\mathrm{div}\vec{F} + \vec{F} \bullet \nabla f$
   (e) $\mathrm{curl}(f\vec{F}) = f\mathrm{curl}\vec{F} + \mathrm{curl}\vec{F} \times \nabla f$ (f) none of the above

6. Let $\vec{v} \in \mathbb{R}^3$ be a constant, non-zero vector. Denote by S the surface of the cube with vertices $(\pm1, \pm1, \pm1)$ with inward-pointing normal vectors. Compute the surface integral $\iint_S 2\vec{v} \bullet \vec{n}dS$.
   (a) $-2|\vec{v}|$ (b) $-|\vec{v}|$ (c) 0 (d) $|\vec{v}|$ (e) $2|\vec{v}|$ (f) $3|\vec{v}|$

PART II: Written Problems. To earn full credit for the following problems you must show your work. You can leave answers in terms of fractions and square roots.

1. Find the point at which the two lines

$$\vec{r}_1(t) = \langle 1, 1, 0\rangle + t\langle 1, -1, 2\rangle, \qquad \vec{r}_2(s) = \langle 2, 0, 2\rangle + s\langle -1, 1, 0\rangle$$

intersect.

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2. Find the work done by the vector field $\vec{F} = x^2y\vec{i} + \frac{1}{3}x^3\vec{j} + xy\vec{k}$ along the curve of intersection of the paraboloid $z = y^2 - x^2$ and the cylinder $x^2 + y^2 = 1$, oriented counterclockwise as viewed from above.

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3. Find every point on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ at which the tangent plane is parallel to the plane $3x - y + 3z = 1$.

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4. Consider the vector field $\vec{F}(x, y, z) = y\vec{i} + (x + z)\vec{j} + y\vec{k}$.
   (a) Is this vector field conservative? If so, find a potential function for $\vec{F}$; if not, explain.
   (b) Compute the line integral of $\vec{F}$ along the line segment from $(2, 1, 4)$ to $(8, 3, -1)$.

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5. Evaluate $\iint_S \vec{F} \bullet d\vec{s}$ where $\vec{F} = \langle -x, -y, z^3\rangle$ and S is the part of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 3$, with downward orientation.